LIMITING REGIMES OF CONDUCTOR ACCELERATION BY ELECTROMAGNETIC FORCES

S. A. Kalikhman

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The acceleration of conductors by electromagnetic forces is a process used in the study of high speed collisions [1, 2]. While heating of the driven body has been considered in sufficient detail [3, 4], mechanical deformations and stresses in the conductor during motion have not been studied sufficiently. In addition, it is clear that deformation of the conductor can not only distort the picture of high-speed interaction, but can also lead to conductor breakage into individual chaotically oriented fragments, acceleration of which is no longer possible. The goal of the present study is to investigate limiting acceleration regimes from the viewpoint of permissible deformations and selection of parameters providing the greatest possible velocity.

We will consider the motion of a conductive elastic bar of length ℓ and mass per unit length m along the axis OZ, perpendicular to the bar axis, udner the action of an electromagnetic load distributed along the bar $F(y, t) = (B + Ay)^2\sin^2 \omega t$, $y \in [0.5\ell; -0.5\ell]$, where A, B are constants. Such a dependence of the accelerating force is characteristic of the electromagnetic accelerators described in [2, 4]. The bar motion can be represented as translation together with a movable coordinate system fixed to the center of mass, and motion relative to that system. Using the theorem of motion of the center of mass and considering deformations and velocities at the initial moment equal to zero, after integration we find the law of motion of the movable coordinate system (center of mass) relative to the fixed system:

$$Z_S = \frac{Al^2 + 12B}{48m\omega^2}(\omega^2 t^2 - \sin^2 \omega t). \tag{1}$$

For thin bars, where the effects of inertia of rotation and transverse shear deformations can be neglected, the conductor motion in the movable coordinate system is described by the elastic oscillation equation [5]:

$$EJ\alpha^{\mathrm{IV}} + m\ddot{\alpha} = f(y, t), \tag{2}$$

where α is the deflection caused by bending deformation; E is the effective modulus of elasticity with consideration of the impulsive character of loading and heating; J is the moment of inertia; $f(y, t) = A(y^2 - \ell^2/12)\sin^2 \omega t$; and initial and boundary conditions are:

$$\alpha(y, 0) = 0,\ \dot{\alpha}(y, 0) = 0,$$
$$\alpha''(l/2, t) = \alpha''(-l/2, t) = 0,\ \alpha'''(l/2, t) = \alpha'''(-l/2, t) = 0$$

(the dot and prime denote derivatives with respect to time t and coordinate y).

Relying on the well-known solution of Eq. (2), presented, for example, in [5], after transformations we find

$$\alpha(\varepsilon, t) = -\frac{Al^6}{EJ}\sum_{i=1}^{\infty}\frac{\mathrm{tg}\,\frac{\xi_{2i-1}}{2}}{\xi_{2i-1}^7}\left(\frac{\mathrm{sh}\,\varepsilon\xi_{2i-1}}{\mathrm{ch}\,\frac{\xi_{2i-1}}{2}} - \frac{\sin \varepsilon\xi_{2i-1}}{\cos\frac{\xi_{2i-1}}{2}}\right)\times \tag{3}$$
$$\times\left(1 + \frac{4\omega_1^2\cos\xi_{2i-1}^2\tau - \xi_{2i-1}^4\cos 2\omega_1\tau}{\xi_{2i-1}^4 - 4\omega_1^2}\right),\quad F(\varepsilon, \tau) = (B + A\varepsilon^2)\sin^2\omega_1\tau,$$

where $\varepsilon = y/x_0$; $\varepsilon \in [-0.5; 0.5]$; $\omega_1 = \omega t_0$; $\tau = t/t_0$; $x_0 = l$; $t_0 = l^2/a$; $a = EJ/m$; ξ_{2i-1} are roots of the equation $\cos \xi_{2i-1} = \cosh^{-1}\xi_{2i-1}$.

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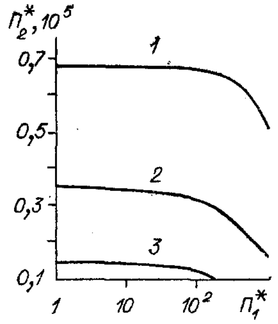

Fig. 1

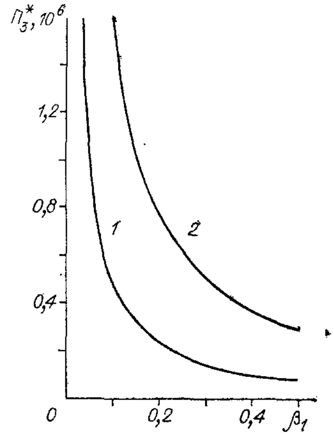

Fig. 2

Depending on the sign of the constant A two electromagnetic load profiles are possible: load at the center of the bar ($\varepsilon = 0$) greater than at the ends, or load at the ends greater than the value at the center. We will study the limiting acceleration regimes for both cases.

1. The Parameter $A = -A_1$, $A_1 \geq 0$. In this case there develop within the bar tensile stress σ_1 due to inflection and tensile forces, and a stress σ_2 produced by the action of the horizontal component of the electromagnetic force y. Following [6], we write

$$\sigma_1 = -\alpha'' Er, \quad \sigma_2 = \frac{1}{S}\int_{-0,5}^{\varepsilon} B(1-\beta_1\varepsilon^2)\frac{\partial\alpha}{\partial\varepsilon}\,d\varepsilon. \tag{4}$$

Here r is the coordinate of the far point of the section $\alpha'' = \frac{1}{l^2}\frac{\partial^2\alpha}{\partial\varepsilon^2}$; $\beta_1 = A_1 l^2/B$; S, is the cross-sectional area. The conditions for bar failure are $\sigma_1 + \sigma_2 = [\sigma]$, where $[\sigma]$ is determined by the mechanical properties of the material with consideration of the impulsive character of loading and heating.

From Eqs. (3), (4) we obtain an equation for calculating the acceleration regime limited by the failure conditions

$$1 - 4\left[\Pi_2^*\beta_1\sin^2\omega_1\tau\int_{-0,5}^{\varepsilon_1}(1-\beta_1\varepsilon^2)\Sigma_{i2}\,d\varepsilon - \Pi_1^*\beta_1\Sigma_{i1}\right] = 0; \tag{5}$$

$$\Pi_1^* = \frac{l^2 rB}{J[\sigma]},\ \Pi_2^* = \frac{B^2 l^4}{EJS[\sigma]},$$

$$\Sigma_{i1} = \sum_{i=1}^{\infty}\frac{\operatorname{tg}\frac{\xi_{2i-1}}{2}}{\xi_{2i-1}^5}\left(\frac{\operatorname{ch}\xi_{2i-1}\varepsilon_1}{\operatorname{ch}\frac{\xi_{2i-1}}{2}} - \frac{\cos\xi_{2i-1}\varepsilon_1}{\cos\frac{\xi_{2i-1}}{2}}\right)f_{2i-1}(\tau),$$

$$\Sigma_{i2} = \sum_{i=1}^{\infty}\frac{\operatorname{tg}\frac{\xi_{2i-1}}{2}}{\xi_{2i-1}^6}\left(\frac{\operatorname{sh}\xi_{2i-1}\varepsilon}{\operatorname{ch}\frac{\xi_{2i-1}}{2}} - \frac{\sin\xi_{2i-1}\varepsilon}{\cos\frac{\xi_{2i-1}}{2}}\right)f_{2i-1}(\tau),$$

$$f_{2i-1}(\tau) = 1 + \left(\xi_{2i-1}^4 - 4\omega_1^2\right)^{-1}\left(4\omega_1^2\cos\xi_{2i-1}^2\tau - \xi_{2i-1}^4\cos 2\omega_1\tau\right).$$

In the case of resonance of one of the frequencies, where $\xi_{2i-1}^2 = 2\omega_1$, the expression for the time cofactor takes on the form

$$f_{2i-1}(\tau) = 1 - (\omega_1\tau\sin 2\omega_1\tau + \cos 2\omega_1\tau).$$

Equation (5) was used to calculate danger parameter curves (Fig. 1, $\omega_1 = 31.4$, $\beta_1 = 0.1$, 0.2, 0.5, lines 1-3) for various frequencies of the accelerating force and degrees of loading force inhomogeneity β_1. The bar coordinate ε_1 and the time within the first semiperiod at which the tensile stresses are greatest were determined. The calculation results show that, except for the case of resonance, the main contribution to bar deformation is produced by the first harmonic. The values of the phase angle $\omega_1\tau$ corresponding to the highest stresses exceed $\pi/2$.

2. Parameter A > 0. In this case the horizontal components of the electromagnetic force which appear due to bar deflection may lead to a loss of stability earlier than failure occurs due to compressive stresses. To determine the critical force value we use the energy method of [6], based on equality of the critical force to the deflection energy. Considering the low value of bending and taking the equation of the elastic line in the form $y = C(1 - \cos\pi\varepsilon)$, after transformations we find for the critical value of the force parameter

$$\Pi_3^* = \left\{\frac{8\sin^2\omega_1\tau}{\pi^2}\int\limits_0^{0,5}\beta_1(1+\beta_1\varepsilon^2)\left(\varepsilon - \frac{\sin 2\pi\varepsilon}{2\pi}\right)\Sigma_{i2}\,d\varepsilon\right\}^{-1} \qquad (6)$$

$$\left(\Pi_3^* = B^2\frac{l^6}{(FJ)^2}\right).$$

Results of calculations with Eq. (6) (Fig. 2, ω_1 = 3.14 and 31.4, lines 1 and 2) permit finding the maximum permissible accelerating forces at which the bar velocity and trajectory can be defined.

In the design of electromagnetic accelerators for solid bodies the fundamental parameters are the required values of conductor velocity (for given dimensions and density) and length of the acceleration path. Using Eq. (1) in relative units, we find an expression for the velocity of the center of mass v_1 and the path traversed, equal to the length of the acceleration channel H_1:

$$v_1 = \frac{Bl^3}{EJ}\frac{12-\beta_1}{48}(2\omega_1\tau_1 - \sin 2\omega_1\tau_1),$$

$$H_1 = \frac{Bl^3}{EJ}\frac{12-\beta_1}{48\omega_1}(\omega_2^2\tau_1^2 - \sin^2\omega_1\tau_1).$$

For the acceleration time τ_1 we have

$$\frac{\omega_1 H_1}{v_1} = \frac{(\omega_1\tau_1)^2 - \sin^2\omega_1\tau_1}{2\omega_1\tau_1 - \sin 2\omega_1\tau_1}. \qquad (7)$$

It was noted earlier that for practically all parameter values the highest stresses develop in the middle of the bar. Thus, in calculating the sums and integrals we may consider $\varepsilon_1 = 0$, which significantly simplifies the calculations. Equation (5) then takes on the form

$$8\beta_1^2\Pi_2^*\Sigma_{i21}\sin\omega_1\tau_1 - 4\beta_1\left(\Pi_2^*\Sigma_{i20}\sin^2\omega_1\tau_1 - \Pi_1^*\Sigma_{i10}\right) + 1 = 0. \qquad (8)$$

Here Σ_{i10} corresponds to the expression for Σ_{i1} in Eq. (5) at $\varepsilon_1 = 0$;

$$\Sigma_{i20} = \sum_{i=1}^{\infty}\frac{\operatorname{tg}\frac{\xi_{2i-1}}{2}}{\xi_{2i-1}^7}\left[\left(\operatorname{ch}\frac{\xi_{2i-1}}{2}\right)^{-1} + \left(\cos\frac{\xi_{2i-1}}{2}\right)^{-1} - 2\right]f_{2i-1}(\tau_1);$$

$$\Sigma_{i21} = \sum_{i=1}^{\infty}\frac{\operatorname{tg}\frac{\xi_{2i-1}}{2}}{\xi_{2i-1}^9}\left[\left(\operatorname{ch}\frac{\xi_{2i-1}}{2}\right)^{-1} - \left(\cos\frac{\xi_{2i-1}}{2}\right)^{-1} - \frac{\xi_{2i-1}^2}{4}\right]f_{2i-1}(\tau_1).$$

Having expressed the force at the center of the bar in terms of the known value of the acceleration channel length

$$B = \frac{48H_1EJ\omega_1^2}{l^3(12-\beta_1)}\left(\omega_1^2\tau_1^2 - \sin^2\omega_1\tau\right)^{-1} \qquad (9)$$

and making use of Eq. (8), we obtain an expression for calculating the permissible load inhomogeneity

$$(8K_1\Sigma_{i21} - 4K_2\Sigma_{i10} + 1)\beta_1^2 - (4K_1\Sigma_{i20} - 48K_2\Sigma_{i10} + 24)\beta_1 + 144 = 0, \tag{10}$$

where
$$K_1 = \frac{2304H_1^2EJ\omega_1^4\sin^2\omega_1\tau_1}{l^2S[\sigma](\omega_1^2\tau_1^2 - \sin^2\omega_1\tau)^2}; \quad K_2 = \frac{48H_1E\omega_1^2\tau_1}{l[\sigma](\omega_1^2\tau_1^2 - \sin^2\omega_1\tau_1)}.$$

Equations (7)-(10) then permit finding admissible values of the force load, with use of which the parameters of the energy source can be determined.

Calculations of limits with respect to mechanical deformation and heating rate (assuming transition of the metal on the conductor axis to the liquid state [3]) show that for degrees of force loading inhomogeneity achieved in practice ($\beta_1 \geq 0.05$) the velocity limitation with respect to mechanical deformation is more stringent than that with regard to heating conditions. Thus, the role of heating reduces to a decrease in strength of the accelerated conductor, as a result of which it fails under the action of mechanical loads.

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